Symbols Representing Arithmetic Operations Solutions

1. If # represents one of the operations +, - and *, is $a\#(b-c)=(a\#b)-(a\#c)$ **for all numbers** a**,** b **and** c.

   (1) $a\#1$ is not equal to $1\#a$ for some numbers a.

   $\#$ is neither addition (as $a+1=1+a$) not multiplication (as $a*1=1*a$), so $\#$ is a subtraction.
   Then $LHS=a\#(b-c)=a-b+c$ and
   $RHS=(a\#b)-(a\#c)=(a-b)-(a-c)=c-b$, so the question becomes
   "is $a-b+c=c-b$ **for all numbers** a**,** b **and** c?" --> "is $a=0$". So when $a=0$ (and $\#$ is a subtraction) then $a\#(b-c)=(a\#b)-(a\#c)$ holds true but not for other values of a, so not **for all numbers** a**,** b **and** c. Answer to the question is NO. Sufficient.

   (2) $\#$ represents subtraction --> the same as above. Sufficient.

   Answer: D.

2. (1) k@1 is not equal to 1@k for some numbers k. @ is neither addition (as $k+1=1+k$) nor multiplication (as $k*1=1*k$), thus @ represents subtraction. Knowing that we can determine whether $k-(l+m)=(k-l)+(k-m)$ *for all numbers k, l,and m*. Sufficient.

   (2) @ represents subtraction. The same here. Sufficient.

   Answer: D.

3. (1)10#5 = 2 --> # can only be division, hence 6#2=6/2=3. Sufficient.

   (2) 4#2 = 2 --> # can be either division or subtraction: if it's division then 6#2=6/2=3 but if it's subtraction then 6#2=6-2=4. Two different answers, not sufficient.

   Answer: A.

4. (1) 2R5 = 10. R can only be multiplication (2*5 = 10), thus 5*2 = 10. Sufficient.

   (2) 5R5 = 25. The same here: R can only be multiplication (5*5 = 25), thus 5*2 = 10. Sufficient.

   Answer: D.

5. (1) 0*2=2 --> operation * represents addition --> 1*0=1+0=1. Sufficient.

   (2) 2*0=2 --> operation * represents either addition or subtraction --> 1*0=1+0=1 and 1*0=1-0=1, the same answer. Sufficient. Answer D.

6. (1) 2 @ k = 3 --> @ can only by addition because if it's multiplication then 2*k=3 -- k=3/2, which is not an integer as stated in the stem. So, we have 2+k=3 --> k=1 --> 3@k=3+1=4. Sufficient.

   (2) 1 @ 0 = k --> @ can be both addition and multiplication, since 1+0=1=k=integer and 1*0=0=k=integer, so we also have two values of k: 1 and 0. In this case 3@k=3+1=4 or 3@k=3*0=0, two different answers. Not sufficient.

   Answer: A.

7. (1) 3#2 > 3, # can be either multiplication or addition in BOTH cases (6#2)#4 = 6#(2#4) is true: (6*2)*4=6*(2*4)=48 and (6+2)+4=6+(2+4)=12. Sufficient.

   (2) 3#1 = 3, # can be either multiplication or division. If it's division the the answer to the question is No and if it's multiplication answer to the question is YES. Two different answers. Not sufficient.

   Answer: A.

8. (1) 0▽1 = 1. The symbol can only be addition. Thus 3 ▽ 2 = 3 + 2 = 5. Sufficient.

   (2) 1▽0 = 1. The symbol is either addition or subtraction. Thus 3 ▽ 2 = 3 + 2 = 5 or 3 ▽ 2 = 3 - 2 = 1. Not sufficient.

   Answer: A.

9. The key here is the bold part of the statement, which tells us that statements MUST be **true for all integers**. I guess your concern is about the statement (2):

   n # n = 0 --> means # can denote only subtraction to be true for ALL integers. Though if n=0 it can denote addition and multiplication as well but one value of n can not determine #.

   So the answer is B.

10. (1) 3#2 > 3. # can be either multiplication or addition. In BOTH cases (6#2)#4 = 6#(2#4) is true: (6*2)*4=6*(2*4)=48 and (6+2)+4=6+(2+4)=12. Sufficient.

    (2) 3#1 = 3. # can be either multiplication or division. If it's division the the answer to the question is No and if it's multiplication answer to the question is YES. Two different answers. Not sufficient.

    Answer: A.

11. Statement-1: 5*1=5*1 multiplication-true
5+1=1+5 addition-true. Eliminate A & D.

Statement-2: 2x * 2x = 4x
It can only be addition. So B is sufficient. Answer is B.

12. (1) a@b=b@a for all numbers a and b --> @ can be addition (a+b=b+a) as well as multiplication (a*b=b*a). Not sufficient.

(2) a@(b-c)=(a@b)-(a@c) for all numbers a, b, and c --> if @ represents addition we will have $a@(b-c)=a+b-c$ which is not equal to $(a@b)-(a@c)=(a+b)-(a+c)=b-c$, so @ must be multiplication. Sufficient. (Just to check: $a@(b-c)=a*(b-c)=ab-ac$ which is equal to $(a@b)-(a@c)=ab-ac$)

Answer: B.

13. The key here is the bold part of the statements, which tells us that statements MUST be **true for all integers**.

(1) n # 0 = n **for all integers n** --> # may denote both addition and subtraction (as n+0=n and n-0=n is true for all integers n), which gives two different values for 1 # 2. Not sufficient.

(2) n # n = 0 **for all integers n** --> # may denote only subtraction to be true for ALL integers (n-n=0 is true for all integers n), though if n=0 it can denote addition and multiplication as well but one value of n can not determine #. So 1 # 2 = 1 - 2 = -1. Sufficient.

Answer: B.

14. (1) x=5. We need to know what § represents.

(2) (−1)§(−2)≠(−2)§(−1). § is neither addition nor multiplication. Thus it's subtraction: (x§2)§x = (x-2)-x=-2. Sufficient.

Answer: B.

15. (1) 5 # 6 = 6 # 5 --> # represents either addition or multiplication in any case (5 # 6) # 2 = 5 # (6 # 2) is true: (5 + 6) + 2 = 5 + (6 + 2) and (5 * 6) * 2 = 5 * (6 * 2). Sufficient.

(2) 2 # 0 = 2 --> # represents either addition or subtraction, if it's addition then the answer is YES (as shown above) but if it's subtraction then the answer is NO: (5 - 6) - 2 =-3 and 5 - (6 - 2) = 1. Not sufficient.

Answer: A.

16. (1) 2 Ω 2 = 4. This implies that Ω represents addition or multiplication (2+2=4 and 2*2=4). If its addition, then 1 Ω 1 = 1 + 1 = 2 but if its multiplication, then 1 Ω 1 = 1*1 = 1. Not sufficient.

    (2) 0 Ω 1 = 0. This implies that Ω represents multiplication or division. In either case 1 Ω 1 = 1. Sufficient.

    Answer: B.

17. The point here is that the question asks whether $(a\mathbb{Q}b)+(a\mathbb{Q}c)=a\mathbb{Q}(b+c)$ is true **FOR ALL NUMBERS a, b, and c**?

    (1) $\mathbb{Q}$ represents subtraction --> the question becomes is $2a-b-c=a-b-c$, or is $a=0$? So $(a\mathbb{Q}b)+(a\mathbb{Q}c)=a\mathbb{Q}(b+c)$ is NOT true for all numbers a, b, and c (so the answer to the question is NO), for this expression to be true a must equal to zero (so not for all values of a). Sufficient.

    (2) $m\mathbb{Q}2\neq2\mathbb{Q}$ --> $\mathbb{Q}$ represents subtraction (as it can not be addition or multiplication), so we have the the same info as above. Sufficient.

    Answer: D.

    Alternately you can see that $(a\mathbb{Q}b)+(a\mathbb{Q}c)=a\mathbb{Q}(b+c)$ to be true **FOR ALL NUMBERS a, b, and c** then $\mathbb{Q}$ must represent multiplication as only for multiplication it's true for all numbers: $ab+ac=ab+ac$. So the question basically ask whether $\mathbb{Q}$ represents multiplication, both (1) and (2) give answer No toth is question.